Supplementary Materials for: Finding and Accounting for Separation Bias in Strategic Choice Models (Not for publication)

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Contents

A Small sample size

The first additional simulation we consider is: how well to the estimators perform with a smaller sample size. This is an important check as separation problems are more common in smaller datasets. Additionally, the biases introduced by small samples can compound separation problems. Finally, one of our main replications has a small sample ($D = 58$) and we want to know if these tools will work as expected in that situation. As in the main text

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Table A.1: Coefficient estimates, standard errors, and root mean-squared error when separation is present in Player B's decision and $D = 50$

Estimator	α_0	α_1	β_0	β_1	RMSE
Ordinary SBI	1.55	-5.90	-3.87	11.35	14.86
	(0.46)	(6.10)	(5.55)	(7.12)	
BR-SBI	1.45	-1.63	-0.79	2.02	2.18
	(0.36)	(1.45)	(0.90)	(1.14)	
Ordinary FIML	1.43	-7.97	-1.92	7.15	18.29
	(0.45)	(15.92)	(2.59)	(1.02)	
BR-FIML	1.45	-1.94	-0.70	1.76	3.01
	(0.37)	(2.50)	(0.80)	(1.17)	
True Values	1.5	-2	-1	2.5	

Note: Average coefficient estimates for each model with standard deviations in parentheses.

we will first consider a situation where separation is only present in B's decision and then a case where separation is present in both decision nodes. The data generating process for this first simulation still follows Figure 3 in the main text.

In the first set of experiments we set $D = 50$ and the parameters are set to values at the bottom of Table A.1. In the second set of experiments we still have $D = 50$, but we set the parameters to the values presented at the bottom of Table A.2. Note that in both experiments we adjust the parameters compared to the simulations presented in the main text. This is done to accommodate the fact that it takes less extreme parameter values to induce separation with a small sample. As noted in the main text, separation is essentially a small-sample problem and that as sample size increases, it is increasingly unlikely to appear as an issue. These adjustments have the additional value of demonstrating that our main conclusions are not driven by specific choices in parameter values. In the first experiment we still only consider cases where the lp-diagnostic finds that the outcome BD is perfectly predicted, while again only considers cases where the lp-diagnostic finds separation associated with the outcomes BD and SQ

In both small-sample experiment we see that the main results still mostly hold up. The BR-SBI now edges out the BR-FIML, but the differences are fairly minor compared to the

Table A.2: Coefficient estimates, standard errors, and root mean-squared error when separation is present at both decision nodes decision and $D = 50$

Estimator	α_0	α_1	β_0	β_1	RMSE
Ordinary SBI	-1.06	20.76	-0.52	8.83	20.10
	(0.42)	(9.06)	(0.41)	(0.47)	
BR-SBI	-1.09	3.02	-0.50	3.43	2.02
	(0.42)	(1.75)	(0.39)	(0.40)	
Ordinary FIML	-1.06	13.58	-0.52	7.77	12.19
	(0.42)	(5.48)	(0.41)	(1.57)	
BR-FIML	-1.08	3.42	-0.40	3.43	2.12
	(0.43)	(1.92)	(0.38)	(0.39)	
True Values	-1	3.5	-0.5	4	

Note: Average coefficient estimates for each model with standard deviations in parentheses.

gaps between the BR and ordinary estimators. Overall, these results provide additional evidence that separation bias can be very problematic for ordinary estimators and that any type of BR should be preferred.

B Cauchy penalty

Here we reconsider the results in Tables 1, 2, A.1, and A.2, but the FIML now uses a Cauchy penalty. This provides an alternative to Jefferys prior that is computationally easier, but less frequently used. However, its use becomes important to us in the replication of Signorino and Tarar (2006) as the Jeffreys prior fails to exist at several of the parameter guess. Note that the first three rows of these tables will roughly, and sometimes exactly, repeat values from previous experiments in order to allow for comparison.

In the first two experiments, which repeat the simulations from the main text we see that the Cauchy penalty term performs similarly to the other BR estimators. It outperforms the Jeffreys prior correction in the first simulation reported in Table B.1, while in the second set performs worse. This poor performance in the experiment reported in Table B.2, appears to be driven by a unusually high variance and is likely a fluke, as it performs very well in every other condition we consider. Overall, it still represents strong improvements over any of the ordinary estimators.

Table B.1: Coefficient estimates, standard errors, and root mean-squared error when separation is present in Player B's decision with $D = 500$ and a Cauchy penalty for the BR-FIML

Estimator	α_0	α_1	β_0	β_1	RMSE
Ordinary SBI	1.50	-2.34	-1.04	9.29	4.60
	(0.13)	(1.45)	(0.45)	(0.53)	
BR-SBI	1.50	-2.04	-1.00	3.83	1.55
	(0.13)	(0.85)	(0.38)	(0.40)	
Ordinary FIML	1.50	-2.43	-1.07	7.14	3.10
	(0.13)	(1.76)	(0.37)	(1.26)	
BR-FIML	1.50	-2.07	-0.96	4.29	1.07
	(0.13)	(0.62)	(0.33)	(0.36)	
True Values	1.5	-2	-1	5	

Note: Average coefficient estimates for each model with standard deviations in parentheses.

The next two sets of experiments consider how well using the Cauchy prior with the BR-FIML performs in the small-sample setting. Once again we see that Cauchy penalty performs very well relative to the alternatives. Unlike the last set of simulations, here the Cauchy-based BR-FIML does well under both cases of separation that we consider: at one decision node and at more than one node.

Table B.2: Coefficient estimates, standard errors, and root mean-squared error when separation is present at both decision nodes with $D = 500$ and a Cauchy penalty for the BR-FIML

Estimator	α_0	α_1	β_0	β_1	RMSE
Ordinary SBI	−3.07	14.79	−0.50	8.87	13.37
	(0.35)	(3.53)	(0.12)	(0.13)	
BR-SBI	−3.07	1.43	−0.50	4.61	1.42
	(0.32)	(0.76)	(0.12)	(0.12)	
Ordinary FIML	−3.06	8.59	−0.50	8.31	7.31
	(0.31)	(2.22)	(0.12)	(0.57)	
BR-FIML	−3.09	1.55	−0.51	5.08	5.89
	(0.35)	(5.75)	(0.22)	(0.67)	
True Values	−3	2.5	−0.5	5	

Note: Average coefficient estimates for each model with standard deviations in parentheses.

C Joint separation induced by a single variable

Next, we consider the effect of having a single variable that induces separation into both actors' utility calculations. Specifically, let B's choice be given by

$$y_B = \mathbb{I}(\underbrace{-0.5 + 0.25X_B + 5X + \varepsilon_B(1)}_{\substack{\text{Expected utility} \\ \text{for } y_B = 1 \mid y_A = 1}} > \underbrace{0 + \varepsilon_B(0)}_{\substack{\text{Expected utility} \\ y_B = 0 \mid y_A = 1}})$$
$$= \mathbb{I}(-0.5 + 0.25X_B + 5X + \varepsilon_B(1) - \varepsilon_B(0) > 0),$$

and A's decision is

$$y_A = \mathbb{I}(\underbrace{(0.25X_A + 2.5X)p_B + \varepsilon_A(1)}_{\substack{\text{Expected utility} \\ \text{for } y_A = 1}} > \underbrace{-2 + \varepsilon_A(0)}_{\substack{\text{Expected utility} \\ \text{for } y_A = 0}})$$
$$= \mathbb{I}(2 + (0.25X_A + 2.5X)p_B + \varepsilon_A(1) - \varepsilon_A(0) > 0).$$

Note that the variable X enters both the utility function for both A and B, while X_A and X_B are unique to their respective actors. Additionally, X_A and X_B are now distributed i.i.d

Table B.3: Coefficient estimates, standard errors, and root mean-squared error when separation is present in Player B's decision with $D = 50$ and a Cauchy penalty for the BR-FIML

Estimator	α_0	α_1	β_0	β_1	RMSE
Ordinary SBI	1.45	−5.46	−3.06	10.59	13.85
	(0.42)	(6.94)	(4.53)	(6.43)	
BR-SBI	1.39	−1.44	−0.70	1.99	1.96
	(0.36)	(1.14)	(0.79)	(1.06)	
Ordinary FIML	1.43	−7.97	−1.92	7.15	18.29
	(0.45)	(15.92)	(2.59)	(4.02)	
BR-FIML	1.43	−1.46	−0.37	1.87	1.85
	(0.36)	(0.89)	(0.72)	(0.93)	
True Values	1.5	−2	−1	2.5	

Note: Average coefficient estimates for each model with standard deviations in parentheses.

Table B.4: Coefficient estimates, standard errors, and root mean-squared error when separation is present at both decision nodes with $D = 50$ and a Cauchy penalty for the BR-FIML

Estimator	α_0	α_1	β_0	β_1	RMSE
Ordinary SBI	−1.06	20.76	−0.52	8.83	20.10
	(0.42)	(9.06)	(0.41)	(0.47)	
BR-SBI	−1.09	3.02	−0.50	3.43	2.02
	(0.42)	(1.75)	(0.39)	(0.40)	
Ordinary FIML	−1.06	13.58	−0.52	7.77	12.19
	(0.42)	(5.48)	(0.41)	(1.57)	
BR-FIML	−1.15	3.10	−0.34	3.70	1.37
	(0.42)	(1.06)	(0.36)	(0.39)	
True Values	−1	3.5	−0.5	4	

Note: Average coefficient estimates for each model with standard deviations in parentheses.

standard normal, while X is distributed Bernoulli with mean 0.5. As in the main simulations we use a sample size of 500 and we only consider the cases where the lp-diagnostic finds separation associated with outcomes SQ and BD. The results are reported in Table C.1, where we see that the results of this slightly more complex situation are largely consistent with the results in the main text. All the BR estimators perform better than their ordinary counterparts, and the BR-FIML estimators (either correction) outperforms the BR-SBI.

Table C.1: Coefficient estimates, standard errors, and root mean-squared error when a single variable produces separation in both players' utilities

Estimator	α_0	α_1	α_2	β_0	β_1	β_2	RMSE
Ordinary SBI	−2.03	0.25	6.99	−0.51	0.25	9.01	6.09
	(0.18)	(0.48)	(0.66)	(0.12)	(0.12)	(0.30)	
BR-SBI	−2.01	0.22	2.18	−0.50	0.25	4.68	0.76
	(0.17)	(0.42)	(0.34)	(0.12)	(0.12)	(0.15)	
Ordinary FIML	−2.03	0.25	5.18	−0.51	0.25	8.31	4.35
	(0.18)	(0.48)	(0.59)	(0.12)	(0.12)	(0.38)	
BR-FIML (Jeffreys)	−2.01	0.22	2.25	−0.50	0.25	4.73	0.72
	(0.17)	(0.42)	(0.35)	(0.12)	(0.12)	(0.15)	
BR-FIML (Cauchy)	−2.03	0.20	2.70	−0.50	0.25	5.24	0.65
	(0.17)	(0.38)	(0.30)	(0.12)	(0.12)	(0.16)	
True Values	−2	0.25	2.5	−0.5	0.25	5	

Note: Average coefficient estimates for each model with standard deviations in parentheses.

D Monte Carlo with multiple covariates

Finally, we consider a more sophisticated simulation experiment. Here we use the data from Signorino and Tarar (2006) to conduct a Monte Carlo experiment based on real-world data. In this simulation we use the BR-FIML estimates from Table 5 to generate new outcome data using the independent variables from Signorino and Tarar (2006). For each Monte Carlo iteration we generate new values of the dependent variable and refit the four models considered in Table 5. As in the main text we only consider the Cauchy implementation of the BR-FIML rather than the Jeffreys prior version as this particular example has such a very poorly behaved objective function that the logged Jeffreys prior penalty is frequently undefined. We repeat this simulation 500 times.

Note that this experiment provides two important extensions over the previous simulations. First, as mentioned, it uses real-world data with many covariates, which provides the most realistic setup of any experiment we consider. Second, it uses a data generating process that is based on private information over outcomes rather than actions. Both of the two empirical replications use this information structure, so considering it in simulations is important.

The results of this simulation are reported in Table D.1, where each cell indicates the RMSE of the proposed estimator relative to the RMSE of the BR-FIML. Cases where this ratio is less than 1 are in bold and indicate that the alternative estimator does a better job than the BR-FIML at estimating this quantity. Note that this occurs in only three of the cases considered (about 5%). The final row presents the multivariate RMSE of each estimator relative to the BR-FIML. As expected the BR-SBI performs reasonably well and much better than the ordinary estimators, but overall the BR-FIML is the best in this experiment.

Table D.1: Relative RMSE of Estimates Compared to BR-FIML

	Ordinary SBI	BR-SBI	Ordinary FIML
U_A(SQ): Const.	3.36	1.85	12.16
U_A(SQ): Tit-for-Tat	2.60	1.22	22.72
U_A(SQ): Firm-Flex	1.70	1.04	16.31
U_A(SQ): Democratic Attacker	2.93	1.51	18.52
U_A(SQ): Year	2.53	**0.93**	18.75
U_A(BD): Const.	1.71	1.23	8.01
U_A(War): Nuclear	1.91	1.32	10.93
U_A(War): Immediate Balance	3.17	1.38	16.12
U_A(War): Short-term Balance	3.52	1.89	13.43
U_A(War): Long-term Balance	2.04	1.04	12.64
U_A(War): Military Alliance	1.88	1.16	14.43
U_A(War): Arms Transfers	**0.91**	**0.66**	6.81
U_B(War): Const.	34.00	1.63	23.85
U_B(War): Nuclear	24.62	1.52	24.05
U_B(War): Immediate Balance	33.05	2.08	21.95
U_B(War): Short-term Balance	28.34	1.75	22.11
U_B(War): Military Alliance	20.41	1.31	16.97
U_B(War): Arms Transfers	22.07	1.52	19.82
U_B(War): Foreign Trade	19.21	1.33	24.27
U_B(War): Stalemate	22.20	1.71	15.83
U_B(War): Democratic Defender	23.37	1.56	17.77
Multivariate RMSE	13.01	1.41	14.60

References

Signorino, Curtis S. and Ahmer Tarar. 2006. "A Unified Theory and Test of Extended Immediate Deterrence." *American Journal of Political Science* 50(3):586–605.